

# A HIGH SPEED BIAXIAL TABLE CONTOUR ERROR CONTROLLER

# Helder B. Lacerda

Federal University of Santa Maria, Manufacturing and Machine Design Department (CT - DFPM), 97105-900 – Santa Maria (RS), Brazil

# Eduardo M. Belo

University of São Paulo, São Carlos Engineering School, Mechanical Engineering Department, 13560-970 - São Carlos (SP), Brazil

**Abstract.** A contour error controller (CEC) works simultaneously with the biaxial table PID controllers, helping them. The calculus of the orthogonal path deviation (contour error) is performed with an additional term in the equation, resulting in a more accurate value and enabling the use of this type of motion controller at higher feedrate. The response results are compared with those of common PIDs and non-corrected CEC in order to analyse the effectiveness of this controller over the system.

*Keywords:* Machine tool; Model; PID; Motion; Controller.

# 1. INTRODUCTION

Koren and Lo (1992) classified the sources of path error deviation in machine tools into three categories: mechanical hardware deficiencies (backlash, non-straightness, etc.), cutting process effects (tool deflection, tool wear, thermal deformation, etc.) and the controller and drive dynamics. The total dimensional error is a combination of the errors from these sources. The first and the second error sources can be minimised by improving the quality of the mechanical hardware or by using compensation techniques. The third set of error sources can be reduced by improving the motion control algorithms. Machine tool builders frequently overlooked this error source, but it can be the dominant one, especially in high speed machining.

Contour error motion controllers were designed in the beginning of the 80's by Koren (1980) to improve machine tool contouring performance by using a different approach than the usual one. While P, PID, state-feedback and feedforward controllers are intended to reduce the axial positioning errors, the contour error controller had the philosophy that its unique objective is the elimination of the contour error. It helps and works together with the machine axial controllers. The problems with this kind of controllers are the necessity of a fast processor to do real time calculations and the lack of accuracy when tracking non-linear contours at high speeds. The first problem is naturally being solved by the evolution of the microprocessors and the latter is the main objective of this paper. An additional term is

introduced in order to make better contour error estimation in high speed contouring operations. Simulations performed with Simulink<sup>®</sup> software (Hicklin, 1992) and actual experimental data support these ideas.

#### 2. AXIS DYNAMIC MODEL

A non-linear dynamic model of a PID controlled biaxial table is used to simulate the system. Each axis consists of a DC motor, ballscrew and carriage. There is Coulomb friction in the ballscrew and guideways. Inertia, backlash and cutting forces are the other disturbance effects. There are thermal deformations in all these elements, but they were not considered here, because the axis temperature variations are supposed small. Guideways and ballscrew pitch errors were also not considered. The idealised linear motion axis model is shown in Figure 1, where  $I_1$  is the sum of the motor and ballscrew inertia,  $I_2$  is the equivalent carriage plus load inertia, K is the combined axis stiffness coefficient and B is the damping coefficient.

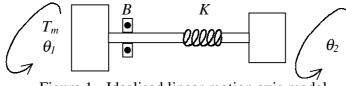


Figure 1 - Idealised linear motion axis model.

The torque from the motor is  $T_m$  and  $\theta_l$  and  $\theta_2$  are the angular displacements at the motor and carriage positions, respectively. The motion equations were obtained by making the torque summation equal to zero. The DC motor equation was taken from Kuo (1985). In the state space form, the following set of equations were obtained, where  $i_a$  is the armature current,  $L_a$  is the electrical inductance,  $R_a$  is the armature electrical resistance,  $K_b$  is an electrical constant,  $v_a$  is the motor applied voltage and  $T_f$  is the Coulomb friction:

$$X(t) = A \cdot X(t) + B \cdot u(t) \tag{1}$$

where:

$$\begin{aligned} X(t) &= \begin{bmatrix} \theta_1 & \omega_1 & \theta_2 & \omega_2 & i_a \end{bmatrix}^T \\ A &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -K.I_1^{-1} & -B.I_1^{-1} & K.I_1^{-1} & B.I_1^{-1} & K_T.I_1^{-1} \\ 0 & 0 & 0 & 1 & 0 \\ K.I_2^{-1} & B.I_2^{-1} & -K.I_2^{-1} & -B.I_2^{-1} & 0 \\ 0 & -K_b.L_a^{-1} & 0 & 0 & -R_a.L_a^{-1} \end{bmatrix} \\ B &= \begin{bmatrix} 0 & 0 & 0 & 0 & L_a^{-1} \\ 0 & -I_1^{-1} & 0 & 0 & 0 \end{bmatrix}^T \end{aligned}$$

$$u(t) = \begin{bmatrix} v_a \\ T_f \end{bmatrix}$$

The output equation is given by eq. (2) where  $\ell$  is the ballscrew pitch (m/rad) and y(t) is the axial displacement.

$$y(t) = \begin{bmatrix} 0 & 0 & \ell & 0 & 0 \end{bmatrix} \cdot X(t)$$
(2)

#### **3. THE CONTOUR ERROR CONTROLLER (CEC)**

The axial controllers and CEC can be seen in the simplified block diagram shown in Figure 2. It consists of two main parts: the contour error mathematical model and the control law, which can be a P, PID, Fuzzy Logic (see Lacerda and Belo, 1996 and 1997a) or another type of controller. Note that there is a coupling between the axes. The interpolator sends reference command signals to each axis that are compared with the axial positions. The resulting axial tracking errors feed the axial controllers, whose function is to move the tool to the reference point R (see Figure 3), thus reducing these errors. The contour error model utilises the interpolator data and error signals to calculate the contour error in real time. For straight paths, the contour error is the PQ segment shown in Figure 3 and is obtained by simple geometric calculation, resulting:

$$PQ = E_x \cdot \sin\alpha - E_y \cdot \cos\alpha \tag{3}$$

where  $E_x$  and  $E_y$  are the axial errors and  $\alpha$  is the angle between the tangent of the instantaneous trajectory and the X axis.

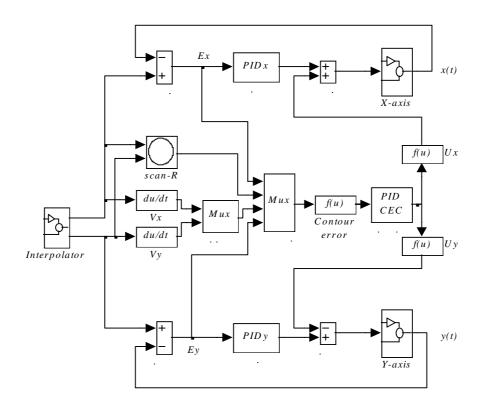


Figure 2 – Simulink<sup>®</sup> block diagram of a biaxial table with CEC.

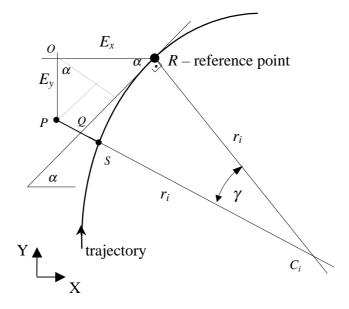


Figure 3 – Axial errors and contour error.

For curved paths, the contour error is the segment *PS*. The segment *QS* can be estimated if we assume that the arc *RS* approximates the hypotenuse of the triangle *POR*. In this case, we can calculate the angle  $\gamma$ , which is a function of the delay between the reference point and the tool position and the instantaneous curvature radius of the trajectory,  $r_i$ :

$$\gamma \cong \frac{\sqrt{E_x^2 + E_y^2}}{r_i} \tag{4}$$

The instantaneous curvature radius  $r_i$  and centre  $C_i$  can be easily calculated by using the positions of three points pertaining to the trajectory, see Lacerda and Belo (1997b). The interpolator supplies these data. In Figure 3, projecting the segment  $QC_i$  in the direction of the perpendicular to the trajectory tangent at reference point R, yields:

$$QC_i \cdot \cos \gamma \cong r_i \tag{5}$$

$$(QS + r_i) \cdot \cos \gamma \cong r_i \tag{6}$$

After a few steps, the segment QS is given by:

or

$$QS \cong r_i . (\sec \gamma - 1) \tag{7}$$

The proposed contour error mathematical model is simply the sum of the segments PQ and QS:

$$\varepsilon \cong E_{\gamma}.sin\alpha - E_{\gamma}.cos\alpha + r_{i}.(sec\gamma - 1)$$
(8)

The contour error is fed into the controller, which sends appropriate correction signals to the individual axes in order to take the tool to point S, in the desired path. In this paper, a PID controller performs the control law.

#### 4. CONTOUR ERROR MODELS TEST

The objective of this simulated test is to compare the proposed contour error mathematical model with three others found in the literature, see Kulkarni and Srinivasan, 1989; Koren and Lo, 1991; Chuang and Liu, 1991. During this test, the axial PID controllers controlled the carriages motions, exclusively. The CEC algorithm was disabled because the interest is on the contour error calculation. The test consists in tracking a 20 mm diameter circle at constant feedrates. The contour error analytical value is obtained by eq.(9), using the distance between two points: the centre of the circle  $(x_c, y_c)$  and the tool position  $(x_p, y_p)$ . This was used as a standard for comparison.

$$\varepsilon = \sqrt{(x_p - x_c)^2 + (y_p - y_c)^2} - R$$
(9)

Initially, a 1 m/min feedrate was chosen to evaluate the model behaviour at a low velocity. The test results (Figure 4) show that all the obtained curves coincided with the analytical curve, thus indicating good accuracy and precision at this velocity.

The feedrate is now increased to 25 m/min and the test results are shown in Figure 5. It can be observed that the Kulkarni and Srinivasan (1989), Koren and Lo (1991) and Chuang and Liu (1991) contour error mathematical models fail in this test, resulting in large discrepancies between the curves. The proposed model curve error still coincided with the analytical error curve. The maximum absolute error, obtained by inspection of the numerical values, is 4  $\mu$ m. This result shows that the proposed contour error mathematical model (eq. 8) is suitable for contour error controllers applied to high speed machine tools.

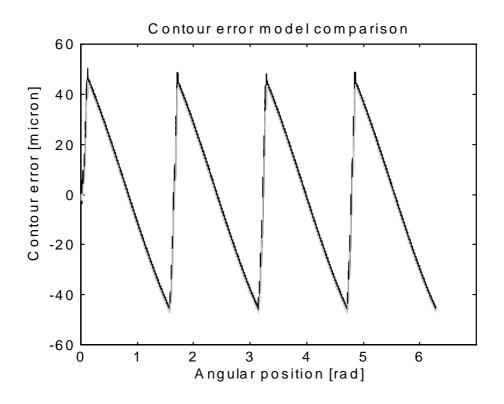
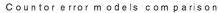


Figure 4 – Contour error mathematical models comparison. Feedrate = 1 m/min



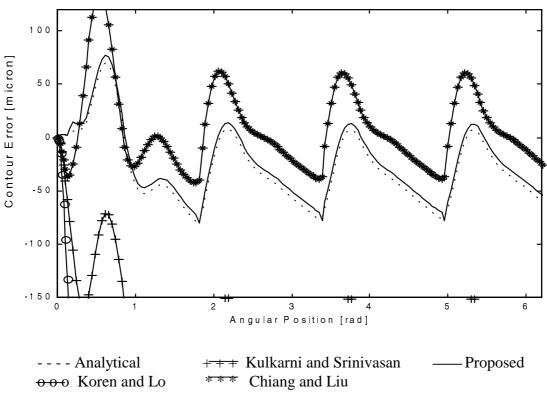


Figure 5 - Contour error mathematical models comparison. Feedrate = 25 m/min.

## **5. SIMULATED CIRCULAR TEST**

The test procedure was initiated by the optimal adjustment of the axial PID controllers gains (see Ästrom and Hägglund, 1995) by using the Matlab<sup>®</sup> optimisation toolbox function FMINS, which uses a Nelder-Mead type simplex search method. The performance criterion is the axial error when tracking a sinusoid. The PID gains are the project variables. The objective is to minimise the axial errors under the constraint that controller output must be in the range  $\pm$  10 volts. After this, the gains of the PID embedded in the CEC were obtained by using the contour error (eq. 9) as the performance criteria with the same constraint as before. This approach resulted in good initial PID gain estimates, but final "manual tuning" was necessary to improve the tracking performance. These gains were not modified during the test. Initially, the CEC algorithm was disabled and the axial PID controllers tracked a 40 mm diameter circle at constant 18.5 m/min feedrate. The results can be viewed in Figure 6 and Table 1, which shows the normalised IAE (integral absolute error) and the maximum circularity error for each test. It is important to note that at this feedrate, the test duration is only 400 ms. After this, the CEC is enabled with the contour error calculated by eq. (3), like in Srinivasan and Kulkarni (1990). The test is performed again, resulting in the curve shown in Figure 6. Finally, the contour error equation is changed to eq. (8) and the circle is tracked again. In Figure 6, there is a magnifying factor of 10x applied to error values. The errors in the first quarter of the circle are bigger because the system starts from rest. It can also be observed a kind of error known as "quadrant glitch" due to friction and inertia. It appears where there is an inversion of the axis motion direction (each  $90^{\circ}$ ).

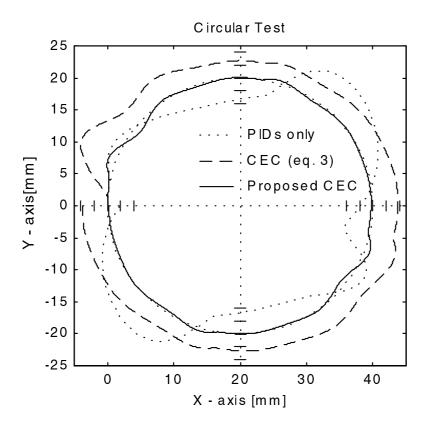


Figure 6 - Comparison between common axial PID and CEC for a circular contour. Circle radius = 20 mm. Feedrate = 18.5 m/min. Magnifying factor: 10x; Marks gap =  $200 \mu m$ .

Table 1 – Normalised IAE and circularity error for the simulated circular test

Controller	IAE	Circularity error [µm]
PID only	1.000	925
CEC eq.(3)	1.325	607
Proposed CEC	0.104	222

#### 6. TEST BENCH DESCRIPTION

The test bench consists of two linear motion axes mounted in the X-Y configuration shown in Figure 7. Each axis is powered by a DC brushless motor, which is fed by a power amplifier. The motor is coupled with a 5 mm pitch ballscrew. An algorithm created with the LabView<sup>®</sup> software<sup>1</sup> running in a PC (personal computer) with a 233 MHz Pentium processor controls the axes. The PC is equipped with an analogue output (AO) board and a counter board. The AO board sends a  $\pm$  10 volts signal to the motor amplifiers. The rotary encoders provide the position feedback signal with 1000 pulses per revolution, resulting in a 5 µm resolution. The trajectory is verified by a KGM 101<sup>®</sup> bidimensional optical encoder<sup>2</sup> which consists of a circular grid plate (160 mm diameter) which carries two line grids

<sup>&</sup>lt;sup>1</sup> LabView is a trademark of National Instruments Corp.

<sup>&</sup>lt;sup>2</sup> KGM 101 is a trademark of Heidenhaim Corp.

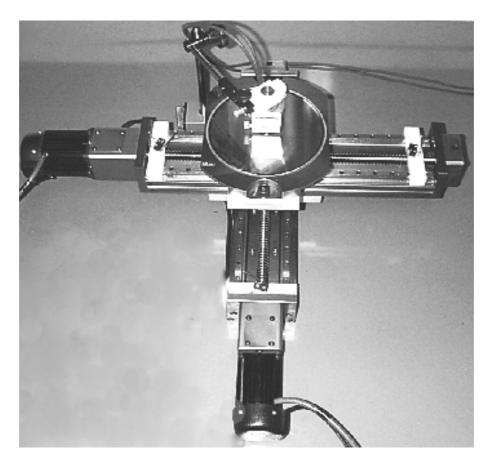


Figure7 - The X-Y table and KGM101<sup>®</sup>.

orthogonal to each other and an optical scanning head, which have two optical reader units disposed 90 degree each other. It simultaneously lights the grid and observes the reflected light. The grid plate is attached to the upper carriage (Y) and the head to the table support. The signal is interpolated in 1024 segments, resulting in a resolution of about 0.1  $\mu$ m. A PC is then used for storing and further processing of the recorded data.

#### 7. EXPERIMENTAL CIRCULAR TEST

The closed-loop (ultimate gain) tuning procedure based on the classical work of Ziegler and Nichols (1942) was used to adjust the gains of the axial PID. The system oscillated a little, but an increase in the derivative time was sufficient to damp it. The adjustment of CEC is more complex, because there is no suitable standard technique to follow in this case. Starting from the PID gains used in the simulated test, a manual tuning was performed which was guided by prior knowledge of the behaviour of the PID controllers. A contour error graph and IAE error criteria displayed on a virtual panel created with the LabView<sup>®</sup> software were of great value in helping to achieve a compromise between minimum error and system stability. A circular test was performed by using the same procedure as was used in the simulation. The results can be seen in Figure 8 and Table 2. They show that the proposed CEC (eq.8) can reduce both the IAE and the circularity error when compared with the system driven only by the PID controllers and by the CEC of eq.(3).

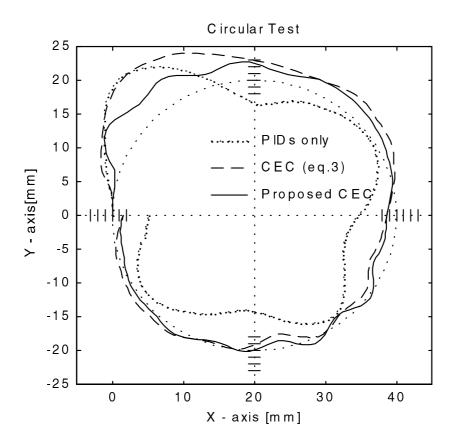


Figure 8 - Comparison between common axial PID and CEC for a circular contour. Circle radius = 20 mm. Feedrate = 18.5 m/min. Magnifying factor: 5 ; Marks gap =  $200 \mu \text{m}$ .

Controller	IAE	Circularity error [µm]
PID only	1.000	2916
CEC eq.(3)	0.629	1983
Proposed CEC	0.396	1218

## 8. CONCLUSION

The main contribution of this paper is the proposed contour error mathematical model. It is simple, accurate and almost insensible to feedrate, thus enabling the use of this type of cross-coupled controllers in high speed machine tools. The simulation results showed that the contour error mathematical models used in earlier controllers are not suitable for high speed machine tools. The experimental results showed a smaller improvement than expected when compared with the other two mentioned controllers. This fact is due to the limitations of the LabView<sup>®</sup> software and the limitations of the PID controller embedded in the CEC. Therefore, the benefits due to the improvement in the contour error estimation when using the proposed mathematical model were partially masked by these physical limitations. A better result can be expected by using a more effective software than LabView<sup>®</sup> and a better controller than the PID. Another contribution of this work is the non-linear, fifth order dynamic XY table simulator, which include the system elasticity and Coulomb friction. It is

able to reproduce well the behaviour of the physical system and was very useful in the development of this and previous works of the authors. **Acknowledgements** 

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### REFERENCES

- Ästrom, K. and T. Hägglund, 1995, PID Controllers: theory, design and tuning, Instrument Society of America, Research Triangle Park, NC, USA.
- Chuang, H. and C. Liu, 1991, Cross-coupled adaptive feedrate control for multiaxis machine tools, ASME J. Dyn. Sys., Meas Control, v. 113, p. 451-457.
- Hicklin, J. et al., 1992, Simulink A program for simulating dynamic systems, The MathWorks Inc., Massachusetts, USA.
- Koren, Y., 1980, Cross-coupled biaxial computer control for manufacturing systems, ASME J. Dyn. Sys., Meas. Control, v. 102, p.265-272.
- Koren, Y. and C. Lo, 1991, Variable-gain cross-coupling controller for contouring, Annals of the CIRP, v. 40, n. 1, p. 371-374.
- Koren, Y. and C. Lo, 1992, Advanced controllers for feed drives, Annals of the CIRP, v. 41, n. 2, p.689-698.
- Kulkarni, P.K. and K. Srinivasan, 1989, Optimal contouring control of multi-axial feed drive servomechanisms, ASME J. of Eng. for Ind., v. 111, p. 140-148.
- Kuo, B., 1985, Automatic Control Systems, Prentice-Hall, Englewoods Cliffs.
- Lacerda, H. B. and E. M. Belo, 1996, Application of fuzzy logic on motion control systems in machine tools, Proceedings of the ICONE'96 - Second International Dynamics, Chaos, Control and their Applications in Conference on Non-linear Engineering Sciences, São Pedro (SP), Brazil, vol.1(1997), p. 301-305.
- Lacerda, H.B. and E. M. Belo, 1997a, Application of a PID-Fuzzy controller on the motion control system in machine tools, Proceedings of the DINAME 97 - 7th International Conference on Dynamic Problems in Mechanics, Angra dos Reis (RJ), Brazil.
- Lacerda, H.B. and E. M. Belo, 1997b, Motion control of a biaxial machine tool using a versatile cross-coupling controller, CD-ROM of the XIV COBEM - Brazilian congress of mechanical engineers (COB221), Bauru (SP), Brazil.
- Srinivasan, K. and P. K. Kulkarni, 1990, Cross-coupled control of biaxial feed drive servomechanisms, ASME J. of Dyn Sys, Meas. and Contr., v. 112, p.225-232.